

CHAPTER 17 (Odd)

3. a. $Z = 15 \Omega - j16 \Omega = 21.93 \Omega \angle -46.85^\circ$
 $E = IZ = (0.5 \text{ A} \angle 60^\circ)(21.93 \Omega \angle -46.85^\circ)$
 $= 10.97 \text{ V} \angle 13.15^\circ$

b. $Z = 10 \Omega \angle 0^\circ \parallel 6 \Omega \angle 90^\circ = 5.15 \Omega \angle 59.04^\circ$
 $E = IZ = (2 \text{ A} \angle 120^\circ)(5.15 \Omega \angle 59.04^\circ)$
 $= 10.30 \text{ V} \angle 179.04^\circ$

5. a. Clockwise mesh currents:

$$\begin{aligned} E - I_1 Z_1 - I_1 Z_2 + I_2 Z_2 &= 0 \\ -I_2 Z_2 + I_1 Z_2 - I_2 Z_3 - E_2 &= 0 \end{aligned}$$

$$\begin{aligned} [Z_1 + Z_2]I_1 - Z_2 I_2 &= E_1 \\ -Z_2 I_1 + [Z_2 + Z_3]I_2 &= -E_2 \end{aligned}$$

$$\begin{aligned} Z_1 &= R_1 \angle 0^\circ = 4 \Omega \angle 0^\circ \\ Z_2 &= X_L \angle 90^\circ = 6 \Omega \angle 90^\circ \\ Z_3 &= X_C \angle -90^\circ = 8 \Omega \angle -90^\circ \\ E_1 &= 10 \text{ V} \angle 0^\circ, E_2 = 40 \text{ V} \angle 60^\circ \end{aligned}$$

$$I_{R_1} = I_1 = \frac{\begin{vmatrix} E_1 & -Z_2 \\ -E_2 & [Z_2 + Z_3] \end{vmatrix}}{\begin{vmatrix} [Z_1 + Z_2] & -Z_2 \\ -Z_2 & [Z_2 + Z_3] \end{vmatrix}} = \frac{[Z_2 + Z_3]E_1 - Z_2 E_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 5.15 \text{ A} \angle -24.5^\circ$$

b. By interchanging the right two branches, the general configuration of part (a) will result and

$$\begin{aligned} I_{50\Omega} = I_1 &= \frac{[Z_2 + Z_3]E_1 - Z_2 E_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} \\ &= 0.442 \text{ A} \angle 143.48^\circ \end{aligned}$$

$$\begin{aligned} Z_1 &= R_1 = 50 \Omega \angle 0^\circ \\ Z_2 &= X_C \angle -90^\circ = 60 \Omega \angle -90^\circ \\ Z_3 &= X_L \angle 90^\circ = 20 \Omega \angle 90^\circ \\ E_1 &= 5 \text{ V} \angle 30^\circ, E_2 = 20 \text{ V} \angle 0^\circ \end{aligned}$$

7. a. Clockwise mesh currents:

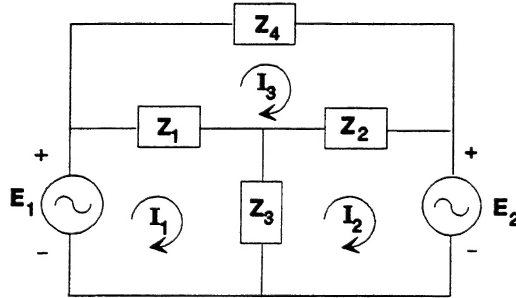
$$\begin{aligned} E_1 - I_1 Z_1 - I_1 Z_2 + I_2 Z_2 &= 0 \\ -I_2 Z_2 + I_1 Z_2 - I_2 Z_3 - I_2 Z_4 + I_3 Z_4 &= 0 \\ -I_3 Z_4 + I_2 Z_4 - I_3 Z_5 - E_2 &= 0 \end{aligned}$$

$$\begin{aligned} Z_1 &= 4 \Omega + j3 \Omega, Z_2 = -j1 \Omega \\ Z_3 &= +j6 \Omega, Z_4 = -j2 \Omega \\ Z_5 &= 8 \Omega \\ E_1 &= 60 \text{ V} \angle 0^\circ, E_2 = 120 \text{ V} \angle 120^\circ \end{aligned}$$

$$\begin{aligned} [Z_1 + Z_2]I_1 - Z_2 I_2 + 0 &= E_1 \\ -Z_2 I_1 + [Z_2 + Z_3 + Z_4]I_2 - Z_4 I_3 &= 0 \\ 0 - Z_4 I_2 + [Z_4 + Z_5]I_3 &= -E_2 \end{aligned}$$

$$\begin{aligned} I_{R_1} = I_3 &= \frac{[Z_2 Z_4]E_1 + [Z_2^2 - [Z_1 + Z_2][Z_2 + Z_3 + Z_4]]E_2}{[Z_1 + Z_2][Z_2 + Z_3 + Z_4][Z_4 + Z_5] - [Z_1 + Z_2]Z_4^2 - [Z_4 + Z_5]Z_2^2} \\ &= 13.07 \text{ A} \angle -33.71^\circ \end{aligned}$$

b.



$$\begin{aligned} Z_1 &= 15 \, \Omega \angle 0^\circ, \quad Z_2 = 15 \, \Omega \angle 0^\circ \\ Z_3 &= -j10 \, \Omega = 10 \, \Omega \angle -90^\circ \\ Z_4 &= 3 \, \Omega + j4 \, \Omega = 5 \, \Omega \angle 53.13^\circ \\ E_1 &= 220 \, \text{V} \angle 0^\circ \\ E_2 &= 100 \, \text{V} \angle 90^\circ \end{aligned}$$

$$\begin{aligned} I_1(Z_1 + Z_3) - I_2Z_3 - I_3Z_1 &= E_1 \\ I_2(Z_2 + Z_3) - I_1Z_3 - I_3Z_2 &= -E_2 \\ I_3(Z_1 + Z_2 + Z_4) - I_1Z_1 - I_2Z_2 &= 0 \end{aligned}$$

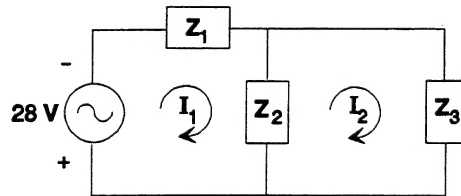
$$\begin{aligned} I_1(Z_1 + Z_3) - I_2Z_3 &\quad - I_3Z_1 &= E_1 \\ -I_1Z_3 &\quad + I_2(Z_2 + Z_3) - I_3Z_2 &= -E_2 \\ -I_1Z_1 &\quad - I_2Z_2 &\quad + I_3(Z_1 + Z_2 + Z_4) = 0 \end{aligned}$$

Applying determinants:

$$\begin{aligned} I_3 &= \frac{-(Z_1 + Z_3)(Z_2)E_2 - Z_1Z_3E_2 + E_1[Z_2Z_3 + Z_1(Z_2 + Z_3)]}{(Z_1 + Z_3)[(Z_2 + Z_3)(Z_1 + Z_2 + Z_4) - Z_2^2] + Z_3[Z_3(Z_1 + Z_2 + Z_4) - Z_1Z_2] - Z_1[-Z_2Z_3 - Z_1(Z_2 + Z_3)]} \\ &= 48.33 \, \text{A} \angle -77.57^\circ \end{aligned}$$

$$\text{or } I_3 = \frac{E_1 - E_2}{Z_4} \text{ if one carefully examines the network!}$$

9.



$$\begin{aligned} Z_1 &= 5 \, \text{k}\Omega \angle 0^\circ \\ Z_2 &= 10 \, \text{k}\Omega \angle 0^\circ \\ Z_3 &= 1 \, \text{k}\Omega + j4 \, \text{k}\Omega = 4.123 \, \text{k}\Omega \angle 75.96^\circ \end{aligned}$$

$$\begin{aligned} I_1(Z_1 + Z_2) - Z_2I_2 &= -28 \, \text{V} \\ I_2(Z_2 + Z_3) - Z_2I_1 &= 0 \end{aligned}$$

$$\begin{aligned} (Z_1 + Z_2)I_1 - Z_2I_2 &= -28 \, \text{V} \\ -Z_2I_1 + (Z_2 + Z_3)I_2 &= 0 \end{aligned}$$

$$I_L = I_2 = \frac{-Z_2 \, 28 \, \text{V}}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} = -3.165 \times 10^{-3} \, \text{V} \angle 137.29^\circ$$

$$\begin{aligned} 11. \quad 6V_x - I_1 \, 1 \, \text{k}\Omega - 10 \, \text{V} \angle 0^\circ &= 0 \\ 10 \, \text{V} \angle 0^\circ - I_2 \, 4 \, \text{k}\Omega - I_2 \, 2 \, \text{k}\Omega &= 0 \end{aligned}$$

$$V_x = I_2 \, 2 \, \text{k}\Omega$$

$$\begin{aligned} -I_1 \, 1 \, \text{k}\Omega + I_2 \, 12 \, \text{k}\Omega &= 10 \, \text{V} \angle 0^\circ \\ -I_2 \, 6 \, \text{k}\Omega &= -10 \, \text{V} \angle 0^\circ \end{aligned}$$

$$I_2 = \frac{10 \text{ V } \angle 0^\circ}{6 \text{ k}\Omega} = 1.667 \text{ mA } \angle 0^\circ = I_{2\text{k}\Omega}$$

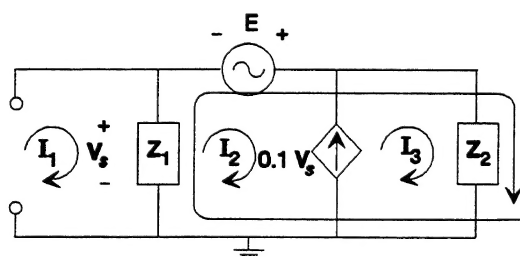
$$-I_1 1 \text{ k}\Omega + (1.667 \text{ mA } \angle 0^\circ)(12 \text{ k}\Omega) = 10 \text{ V } \angle 0^\circ$$

$$-I_1 1 \text{ k}\Omega + 20 \text{ V } \angle 0^\circ = 10 \text{ V } \angle 0^\circ$$

$$-I_1 1 \text{ k}\Omega = -10 \text{ V } \angle 0^\circ$$

$$I_1 = \frac{10 \text{ V } \angle 0^\circ}{1 \text{ k}\Omega} = 10 \text{ mA } \angle 0^\circ$$

13.



$$Z_1 = 1 \text{ k}\Omega \angle 0^\circ$$

$$Z_2 = 4 \text{ k}\Omega + j6 \text{ k}\Omega$$

$$E = 10 \text{ V } \angle 0^\circ$$

$$-Z_1(I_2 - I_1) + E - I_3 Z_2 = 0$$

$$I_1 = 6 \text{ mA } \angle 0^\circ, 0.1 V_s = I_3 - I_2, V_s = (I_1 - I_2)Z_1$$

Substituting:

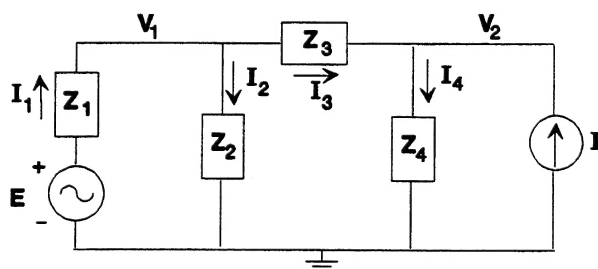
$$(1 \text{ k}\Omega)I_2 + (4 \text{ k}\Omega + j6 \text{ k}\Omega)I_3 = 16 \text{ V } \angle 0^\circ$$

$$(99 \Omega)I_2 + I_3 = 0.6 \text{ V } \angle 0^\circ$$

Determinants:

$$I_3 = I_{6 \text{ k}\Omega(2)} = 1.378 \text{ mA } \angle -56.31^\circ$$

15. a.



$$Z_1 = 5 \Omega \angle 0^\circ$$

$$Z_2 = 6 \Omega \angle 90^\circ$$

$$Z_3 = 4 \Omega \angle -90^\circ$$

$$Z_4 = 2 \Omega \angle 0^\circ$$

$$E = 30 \text{ V } \angle 50^\circ$$

$$I = 0.04 \text{ A } \angle 90^\circ$$

$$I_1 = I_2 + I_3$$

$$\frac{E_1 - V_1}{Z_1} = \frac{V_1}{Z_2} + \frac{(V_1 - V_2)}{Z_3} \Rightarrow V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] - \frac{V_2}{Z_3} = \frac{E_1}{Z_1}$$

$$\text{or } V_1[Y_1 + Y_2 + Y_3] - Y_3 V_2 = E_1 Y_1$$

$$I_3 + I = I_4$$

$$\frac{V_1 - V_2}{Z_3} + I = \frac{V_2}{Z_4} \Rightarrow V_2 \left[\frac{1}{Z_3} + \frac{1}{Z_4} \right] - \frac{V_1}{Z_3} = I$$

$$\text{or } V_2[Y_3 + Y_4] - V_1 Y_3 = I$$

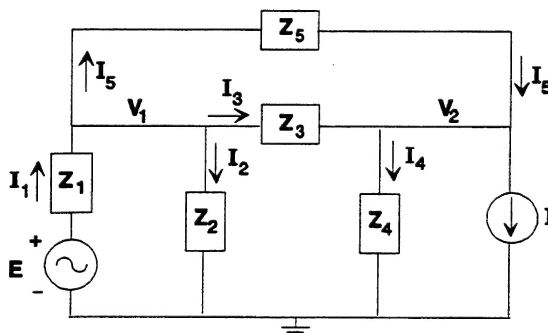
resulting in

$$\begin{aligned} V_1[Y_1 + Y_2 + Y_3] - V_2Y_3 &= E_1Y_1 \\ -V_1[Y_3] + V_2[Y_3 + Y_4] &= +I \end{aligned}$$

Using determinants:

$$V_1 = 19.86 \text{ V } \angle 43.8^\circ \text{ and } V_2 = 8.94 \text{ V } \angle 106.9^\circ$$

b.



$$\begin{aligned} Z_1 &= 10 \Omega \angle 0^\circ \\ Z_2 &= 10 \Omega \angle 0^\circ \\ Z_3 &= 4 \Omega \angle 90^\circ \\ Z_4 &= 2 \Omega \angle 0^\circ \\ Z_5 &= 8 \Omega \angle -90^\circ \\ E &= 50 \text{ V } \angle 120^\circ \\ I &= 0.8 \text{ A } \angle 70^\circ \end{aligned}$$

$$I_1 = I_2 + I_5$$

$$\frac{E - V_1}{Z_1} = \frac{V_1}{Z_2} + \frac{(V_1 - V_2)}{Z_5} + \frac{V_1 - V_2}{Z_3} \Rightarrow V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_5} \right] - V_2 \left[\frac{1}{Z_3} + \frac{1}{Z_5} \right] = \frac{E}{Z_1}$$

$$\text{or } V_1[Y_1 + Y_2 + Y_3 + Y_5] - V_2[Y_3 + Y_5] = E_1Y_1$$

$$I_3 + I_5 = I_4 + I$$

$$\frac{V_1 - V_2}{Z_3} + \frac{V_1 - V_2}{Z_5} = \frac{V_2}{Z_4} + I \Rightarrow V_2 \left[\frac{1}{Z_3} + \frac{1}{Z_4} + \frac{1}{Z_5} \right] - V_1 \left[\frac{1}{Z_3} + \frac{1}{Z_5} \right] = -I$$

$$\text{or } V_2[Y_3 + Y_4 + Y_5] - V_1[Y_3 + Y_5] = -I$$

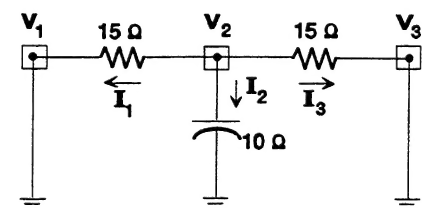
resulting in

$$\begin{aligned} V_1[Y_1 + Y_2 + Y_3 + Y_5] - V_2[Y_3 + Y_5] &= E_1Y_1 \\ -V_1[Y_3 + Y_5] + V_2[Y_3 + Y_4 + Y_5] &= -I \end{aligned}$$

Applying determinants:

$$V_1 = 19.78 \text{ V } \angle 132.48^\circ \text{ and } V_2 = 13.37 \text{ V } \angle 98.78^\circ$$

17.



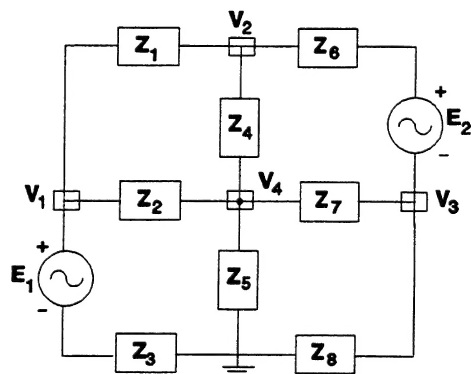
(Note that $3 + j4$ branch has no effect on nodal voltages)

$$\begin{aligned} \sum I_i &= \sum I_o \\ 0 &= I_1 + I_2 + I_3 \\ &= \frac{V_2 - V_1}{15 \Omega} + \frac{V_2}{10 \Omega \angle -90^\circ} + \frac{V_2 - V_3}{15 \Omega} \end{aligned}$$

Through manipulation:

$$\begin{aligned} V_2[2 + j1.5] - V_1 - V_3 &= 0 \\ \text{but } V_1 &= 220 \text{ V } \angle 0^\circ \text{ and } V_3 = 100 \text{ V } \angle 90^\circ \\ \text{and } V_2 &= \frac{220 + j100}{2 + j1.5} = 96.664 \text{ V } \angle -12.426^\circ \end{aligned}$$

19.



$$E_1 = 25 \text{ V } \angle 0^\circ$$

$$E_2 = 75 \text{ V } \angle 20^\circ$$

$$Z_1 = 10 \Omega + j20 \Omega$$

$$Z_2 = 6 \Omega \angle 0^\circ$$

$$Z_3 = 5 \Omega \angle 0^\circ$$

$$Z_4 = 20 \Omega \angle -90^\circ$$

$$Z_5 = 10 \Omega \angle 0^\circ$$

$$Z_6 = 80 \Omega \angle 0^\circ$$

$$Z_7 = 15 \Omega \angle 90^\circ$$

$$Z_8 = 5 \Omega - j20 \Omega$$

$$V_1: \frac{V_1 - V_2}{Z_1} + \frac{V_1 - V_4}{Z_2} + \frac{V_1 - E_1}{Z_3} = 0$$

$$V_2: \frac{V_2 - V_1}{Z_1} + \frac{V_2 - V_4}{Z_4} + \frac{V_2 - E_2 - V_3}{Z_6} = 0$$

$$V_3: \frac{V_3 + E_2 - V_2}{Z_6} + \frac{V_3 - V_4}{Z_7} + \frac{V_3}{Z_8} = 0$$

$$V_4: \frac{V_4 - V_1}{Z_2} + \frac{V_4 - V_2}{Z_4} + \frac{V_4 - V_3}{Z_7} + \frac{V_4}{Z_5} = 0$$

Rearranging:

$$V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right] - \frac{V_2}{Z_1} - \frac{V_4}{Z_2} = \frac{E_1}{Z_3}$$

$$V_2 \left[\frac{1}{Z_1} + \frac{1}{Z_4} + \frac{1}{Z_6} \right] - \frac{V_1}{Z_1} - \frac{V_4}{Z_4} - \frac{V_3}{Z_6} = \frac{E_2}{Z_6}$$

$$V_3 \left[\frac{1}{Z_6} + \frac{1}{Z_7} + \frac{1}{Z_8} \right] - \frac{V_2}{Z_6} - \frac{V_4}{Z_7} = -\frac{E_2}{Z_6}$$

$$V_4 \left[\frac{1}{Z_2} + \frac{1}{Z_4} + \frac{1}{Z_7} + \frac{1}{Z_5} \right] - \frac{V_1}{Z_2} - \frac{V_2}{Z_4} - \frac{V_3}{Z_7} = 0$$

Setting up and then using determinants:

$$V_1 = 14.62 \text{ V } \angle -5.861^\circ, V_2 = 35.03 \text{ V } \angle -37.69^\circ$$

$$V_3 = 32.4 \text{ V } \angle -73.34^\circ, V_4 = 5.667 \text{ V } \angle 23.53^\circ$$

21. Left node:

$$V_1$$

$$\sum I_i = \sum I_o$$

$$4I_x = I_x + 5 \text{ mA } \angle 0^\circ + \frac{V_1 - V_2}{2 \text{ k}\Omega}$$

Right node:

$$V_2$$

$$\sum I_i = \sum I_o$$

$$8 \text{ mA } \angle 0^\circ = \frac{V_2}{1 \text{ k}\Omega} + \frac{V_2 - V_1}{2 \text{ k}\Omega} + 4I_x$$

$$\text{Insert } I_x = \frac{V_1}{4 \text{ k}\Omega \angle -90^\circ}$$

Rearrange, reduce and 2 equations with 2 unknowns result:

$$\begin{aligned} V_1[1.803 \angle 123.69^\circ] + V_2 &= 10 \\ V_1[2.236 \angle 116.57^\circ] + 3 V_2 &= 16 \end{aligned}$$

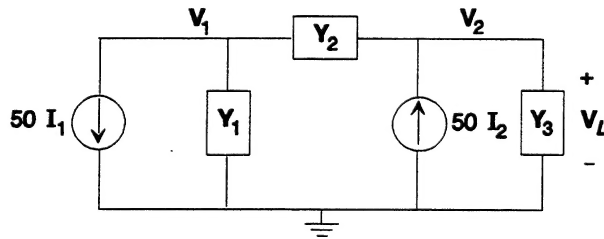
$$\begin{aligned} \text{Determinants: } V_1 &= 4.372 \text{ V } \angle -128.655^\circ \\ V_2 &= 2.253 \text{ V } \angle 17.628^\circ \end{aligned}$$

23. Left node: V_1
 $\sum I_i = \sum I_o$
 $2 \text{ mA } \angle 0^\circ = 12 \text{ mA } \angle 0^\circ + \frac{V_1}{2 \text{ k}\Omega} + \frac{V_1 - V_2}{1 \text{ k}\Omega}$
 and $1.5 V_1 - V_2 = -10$

Right node: V_2
 $\sum I_i = \sum I_o$
 $0 = 2 \text{ mA } \angle 0^\circ + \frac{V_2 - V_1}{1 \text{ k}\Omega} - \frac{V_2 - 6 \text{ V}_x}{3.3 \text{ k}\Omega}$
 and $2.7 V_1 - 3.7 V_2 = -6.6$

$$\begin{aligned} \text{Using determinants: } V_1 &= -10.667 \text{ V } \angle 0^\circ = 10.667 \text{ V } \angle 180^\circ \\ V_2 &= -6 \text{ V } \angle 0^\circ = 6 \text{ V } \angle 180^\circ \end{aligned}$$

25.



$$\begin{aligned} I_1 &= \frac{E_i \angle \theta}{R_1 \angle 0^\circ} = 1 \times 10^{-3} E_i \\ Y_1 &= \frac{1}{50 \text{ k}\Omega} = 0.02 \text{ mS } \angle 0^\circ \\ Y_2 &= \frac{1}{1 \text{ k}\Omega} = 1 \text{ mS } \angle 0^\circ \\ Y_3 &= 0.02 \text{ mS } \angle 0^\circ \\ I_2 &= (V_1 - V_2)Y_2 \end{aligned}$$

$$\begin{aligned} V_1(Y_1 + Y_2) - Y_2 V_2 &= -50I_1 \\ V_2(Y_2 + Y_3) - Y_2 V_1 &= 50I_2 = 50(V_1 - V_2)Y_2 = 50Y_2 V_1 - 50Y_2 V_2 \end{aligned}$$

$$\begin{aligned} (Y_1 + Y_2)V_1 - Y_2 V_2 &= -50I_1 \\ -51Y_2 V_1 + (51Y_2 + Y_3)V_2 &= 0 \end{aligned}$$

$$V_L = V_2 = \frac{-(50)(51)Y_2 I_1}{(Y_1 + Y_2)(51Y_2 + Y_3) - 51Y_2^2} = -2451.92 E_i$$

27. a. $\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$

$$\frac{4 \times 10^3 \angle 0^\circ}{4 \times 10^3 \angle 90^\circ} \stackrel{?}{=} \frac{4 \times 10^3 \angle 0^\circ}{4 \times 10^3 \angle -90^\circ}$$

$$1 \angle -90^\circ \neq 1 \angle 90^\circ \text{ (not balanced)}$$

b. The solution to 26(b) resulted in

$$I_3 = I_{X_C} = \frac{E(Z_1 Z_5 + Z_3(Z_1 + Z_2 + Z_5))}{Z_\Delta}$$

where $Z_\Delta = (Z_1 + Z_3 + Z_6)[(Z_1 + Z_2 + Z_5)(Z_3 + Z_4 + Z_5) - Z_5^2]$
 $- Z_1[Z_1(Z_3 + Z_4 + Z_5) - Z_3 Z_5] - Z_3[Z_1 Z_5 + Z_3(Z_1 + Z_2 + Z_5)]$
 and $Z_1 = 5 \text{ k}\Omega \angle 0^\circ$, $Z_2 = 8 \text{ k}\Omega \angle 0^\circ$, $Z_3 = 2.5 \text{ k}\Omega \angle 90^\circ$
 $Z_4 = 4 \text{ k}\Omega \angle 90^\circ$, $Z_5 = 5 \text{ k}\Omega \angle -90^\circ$, $Z_6 = 1 \text{ k}\Omega \angle 0^\circ$
 and $I_{X_C} = 1.76 \text{ mA} \angle -71.54^\circ$

c. The solution to 26(c) resulted in

$$V_3 = V_{X_C} = \frac{I[Y_1 Y_5 + Y_2(Y_1 + Y_3 + Y_5)]}{Y_\Delta}$$

where $Y_\Delta = (Y_1 + Y_2 + Y_6)[(Y_1 + Y_3 + Y_5)(Y_2 + Y_4 + Y_5) - Y_5^2]$
 $- Y_1[Y_1(Y_2 + Y_4 + Y_5) + Y_2 Y_5]$
 $- Y_2[Y_1 Y_5 + Y_2(Y_1 + Y_3 + Y_5)]$
 with $Y_1 = 0.2 \text{ mS} \angle 0^\circ$, $Y_2 = 0.125 \text{ mS} \angle 0^\circ$, $Y_3 = 0.4 \text{ mS} \angle -90^\circ$
 $Y_4 = 0.25 \text{ mS} \angle -90^\circ$, $Y_5 = 0.2 \text{ mS} \angle 90^\circ$

Source conversion: $Y_6 = 1 \text{ mS} \angle 0^\circ$, $I = 10 \text{ mA} \angle 0^\circ$
 and $V_3 = 7.03 \text{ V} \angle -18.46^\circ$

29. $X_{C_1} = \frac{1}{\omega C} = \frac{1}{(1000 \text{ rad/s})(3 \text{ }\mu\text{F})} = \frac{1}{3} \text{ k}\Omega$

$$Z_1 = R_1 \parallel X_{C_1} \angle -90^\circ = (2 \text{ k}\Omega \angle 0^\circ) \parallel \frac{1}{3} \text{ k}\Omega \angle -90^\circ = 328.8 \Omega \angle -80.54^\circ$$

$$Z_2 = R_2 \angle 0^\circ = 0.5 \text{ k}\Omega \angle 0^\circ, Z_3 = R_3 \angle 0^\circ = 4 \text{ k}\Omega \angle 0^\circ$$

$$Z_4 = R_x + jX_{L_x} = 1 \text{ k}\Omega + j6 \text{ k}\Omega$$

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

$$\frac{328.8 \Omega \angle -80.54^\circ}{4 \text{ k}\Omega \angle 0^\circ} \stackrel{?}{=} \frac{0.5 \text{ k}\Omega \angle 0^\circ}{6.083 \Omega \angle 80.54^\circ}$$

$$82.2 \angle -80.54^\circ \stackrel{?}{=} 82.2 \angle -80.54^\circ \text{ (balanced)}$$

31. For balance:

$$R_1(R_x + jX_{L_x}) = R_2(R_3 + jX_{L_3})$$

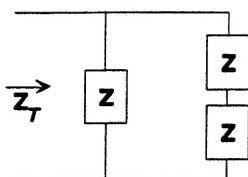
$$R_1R_x + jR_1X_{L_x} = R_2R_3 + jR_2X_{L_3}$$

$$\therefore R_1R_x = R_2R_3 \text{ and } R_x = \frac{R_2R_3}{R_1}$$

$$R_1X_{L_x} = R_2X_{L_3} \text{ and } R_1\omega L_x = R_2\omega L_3$$

$$\text{so that } L_x = \frac{R_2L_3}{R_1}$$

33. a.



$$Z_{\Delta} = 3Z_Y = 3(3 \Omega \angle 90^\circ) = 9 \Omega \angle 90^\circ$$

$$Z = 9 \Omega \angle 90^\circ \parallel (12 \Omega - j16 \Omega)$$

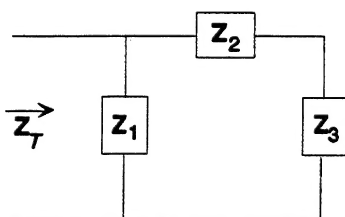
$$= 9 \Omega \angle 90^\circ \parallel 20 \Omega \angle 53.13^\circ$$

$$= 12.96 \Omega \angle 67.13^\circ$$

$$Z_T = Z \parallel 2Z = \frac{2Z^2}{Z + 2Z} = \frac{2}{3}Z = \frac{2}{3}[12.96 \Omega \angle 67.13^\circ] = 8.64 \Omega \angle 67.13^\circ$$

$$I = \frac{E}{Z_T} = \frac{100 \text{ V} \angle 0^\circ}{8.64 \Omega \angle 67.13^\circ} = 11.57 \text{ A} \angle -67.13^\circ$$

b. $Z_{\Delta} = 3Z_Y = 3(5 \Omega) = 15 \Omega$



$$Z_1 = 15 \Omega \angle 0^\circ \parallel 5 \Omega \angle -90^\circ$$

$$= 4.74 \Omega \angle -71.57^\circ$$

$$Z_2 = 15 \Omega \angle 0^\circ \parallel 6 \Omega \angle 90^\circ$$

$$= 5.57 \Omega \angle 68.2^\circ = 2.07 \Omega + j5.17 \Omega$$

$$Z_3 = Z_1 = 4.74 \Omega \angle -71.57^\circ$$

$$= 1.5 \Omega - j4.5 \Omega$$

$$\begin{aligned} Z_T &= Z_1 \parallel (Z_2 + Z_3) = (4.74 \Omega \angle -71.57^\circ) \parallel (2.07 \Omega + j5.17 \Omega + 1.5 \Omega - j4.5 \Omega) \\ &= (4.74 \Omega \angle -71.57^\circ) \parallel (3.63 \Omega \angle 10.63^\circ) \\ &= 2.71 \Omega \angle -23.87^\circ \end{aligned}$$

$$I = \frac{E}{Z_T} = \frac{100 \text{ V} \angle 0^\circ}{2.71 \Omega \angle -23.87^\circ} = 36.9 \text{ A} \angle 23.87^\circ$$

CHAPTER 17 (Even)

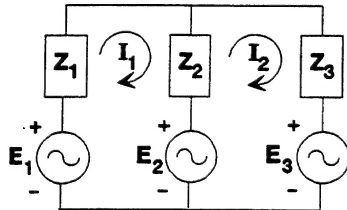
2. a. $Z = 5.6 \Omega + j8.2 \Omega = 9.93 \Omega \angle 55.67^\circ$
 $I = \frac{E}{Z} = \frac{20 \text{ V} \angle 20^\circ}{9.93 \Omega \angle 55.67^\circ} = 2.014 \text{ A} \angle -35.67^\circ$

b. $Z = 2 \Omega \angle 0^\circ \parallel 5 \Omega \angle 90^\circ = 1.86 \Omega \angle 21.8^\circ$
 $I = \frac{E}{Z} = \frac{60 \text{ V} \angle 30^\circ}{1.86 \Omega \angle 21.8^\circ} = 32.26 \text{ A} \angle 8.2^\circ$

4. a. $I = \frac{\mu\text{V}}{R} = \frac{16 \text{ V}}{4 \times 10^3} = 4 \times 10^{-3} \text{ V}$
 $Z = 4 \text{ k}\Omega \angle 0^\circ$

b. $V = (hI)(R) = (50 \text{ I})(50 \text{ k}\Omega) = 2.5 \times 10^6 \text{ I}$
 $Z = 50 \text{ k}\Omega \angle 0^\circ$

6. a.



$$\begin{aligned} Z_1 &= 12 \Omega + j12 \Omega = 16.971 \Omega \angle 45^\circ \\ Z_2 &= 3 \Omega \angle 0^\circ \\ Z_3 &= -j1 \Omega \\ E_1 &= 20 \text{ V} \angle 50^\circ \\ E_2 &= 60 \text{ V} \angle 70^\circ \\ E_3 &= 40 \text{ V} \angle 0^\circ \end{aligned}$$

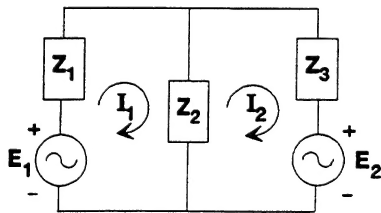
$$\begin{aligned} I_1[Z_1 + Z_2] - Z_2 I_2 &= E_1 - E_2 \\ I_2[Z_2 + Z_3] - Z_2 I_1 &= E_2 - E_3 \end{aligned}$$

$$\begin{aligned} (Z_1 + Z_2)I_1 - Z_2 I_2 &= E_1 - E_2 \\ -Z_2 I_1 + (Z_2 + Z_3)I_2 &= E_2 - E_3 \end{aligned}$$

Using determinants:

$$I_{R_1} = I_1 = \frac{(E_1 - E_2)(Z_2 + Z_3) + Z_2(E_2 - E_3)}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 2.552 \text{ A} \angle 132.72^\circ$$

b.



Source conversion:

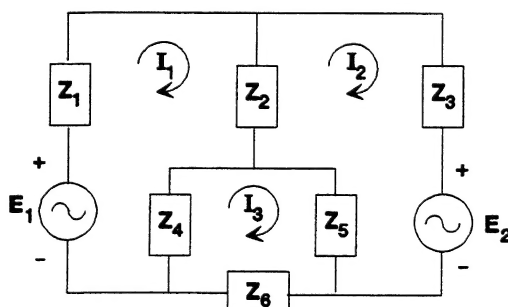
$$\begin{aligned} E_1 &= IZ = (6 \text{ A} \angle 0^\circ)(2 \Omega \angle 0^\circ) \\ &= 12 \text{ V} \angle 0^\circ \\ Z_1 &= 2 \Omega + 20 \Omega + j20 \Omega = 22 \Omega + j20 \Omega \\ &= 29.732 \Omega \angle 42.274^\circ \\ Z_2 &= -j10 \Omega = 10 \Omega \angle -90^\circ \\ Z_3 &= 10 \Omega \angle 0^\circ \end{aligned}$$

$$\begin{aligned} I_1[Z_1 + Z_2] - Z_2 I_2 &= E_1 \\ I_2[Z_2 + Z_3] - Z_2 I_1 &= -E_2 \end{aligned}$$

$$\begin{aligned} (Z_1 + Z_2)I_1 - Z_2 I_2 &= E_1 \\ -Z_2 I_1 + (Z_2 + Z_3)I_2 &= -E_2 \end{aligned}$$

$$I_{R1} = I_1 = \frac{E_1(Z_2 + Z_3) - Z_2 E_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = 0.495 \text{ A } \angle 72.255^\circ$$

8. a.



$$\begin{aligned} Z_1 &= 5 \Omega \angle 0^\circ, Z_2 = 5 \Omega \angle 90^\circ \\ Z_3 &= 4 \Omega \angle 0^\circ, Z_4 = 6 \Omega \angle -90^\circ \\ Z_5 &= 4 \Omega \angle 0^\circ, Z_6 = 6 \Omega + j8 \Omega \\ E_1 &= 20 \text{ V } \angle 0^\circ, E_2 = 40 \text{ V } \angle 60^\circ \end{aligned}$$

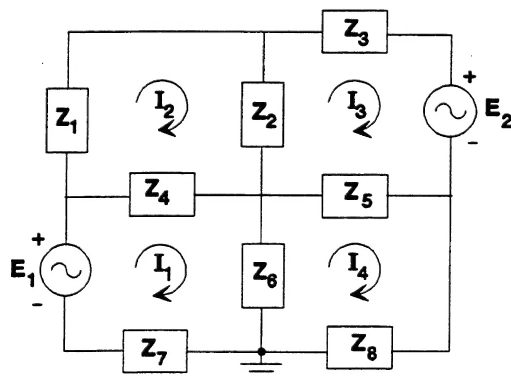
$$\begin{aligned} I_1(Z_1 + Z_2 + Z_4) - I_2 Z_2 - I_3 Z_4 &= E_1 \\ I_2(Z_2 + Z_3 + Z_5) - I_1 Z_2 - I_3 Z_5 &= -E_2 \\ I_3(Z_4 + Z_5 + Z_6) - I_1 Z_4 - I_2 Z_5 &= 0 \end{aligned}$$

$$\begin{aligned} (Z_1 + Z_2 + Z_4)I_1 &\quad - Z_2 I_2 &\quad - Z_4 I_3 &= E_1 \\ -Z_2 I_1 &+ (Z_2 + Z_3 + Z_5)I_2 &\quad - Z_5 I_3 &= -E_2 \\ -Z_4 I_1 &\quad - Z_5 I_2 &+ (Z_4 + Z_5 + Z_6)I_3 &= 0 \end{aligned}$$

Using $Z' = Z_1 + Z_2 + Z_4$, $Z'' = Z_2 + Z_3 + Z_5$, $Z''' = Z_4 + Z_5 + Z_6$
and determinants:

$$\begin{aligned} I_{R1} = I_1 &= \frac{E_1(Z''Z''' - Z_5^2) - E_2(Z_2Z''' + Z_4Z_5)}{Z'(Z''Z''' - Z_5^2) - Z_2(Z_2Z''' + Z_4Z_5) - Z_4(Z_2Z_5 + Z_4Z''')} \\ &= 3.04 \text{ A } \angle 169.12^\circ \end{aligned}$$

b.



$$\begin{aligned} Z_1 &= 10 \Omega + j20 \Omega & Z_2 &= -j20 \Omega \\ Z_3 &= 80 \Omega \angle 0^\circ & Z_4 &= 6 \Omega \angle 0^\circ \\ Z_5 &= 15 \Omega \angle 90^\circ & Z_6 &= 10 \Omega \angle 0^\circ \\ Z_7 &= 5 \Omega \angle 0^\circ & Z_8 &= 5 \Omega - j20 \Omega \\ E_1 &= 25 \text{ V } \angle 0^\circ & E_2 &= 75 \text{ V } \angle 20^\circ \end{aligned}$$

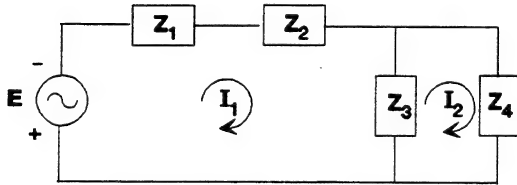
$$\begin{aligned}
 I_1(Z_4 + Z_6 + Z_7) - I_2Z_4 - I_4Z_6 &= E_1 \\
 I_2(Z_1 + Z_2 + Z_4) - I_1Z_4 - I_3Z_2 &= 0 \\
 I_3(Z_2 + Z_3 + Z_5) - I_2Z_2 - I_4Z_5 &= -E_2 \\
 I_4(Z_5 + Z_6 + Z_8) - I_1Z_6 - I_3Z_5 &= 0
 \end{aligned}$$

$$\begin{array}{ccccccc}
 (Z_4 + Z_6 + Z_7)I_1 & & -Z_4I_2 & & +0 & & -Z_6I_4 = E_1 \\
 -Z_4I_1 & + (Z_1 + Z_2 + Z_4)I_2 & & -Z_2I_3 & & +0 & = 0 \\
 0 & -Z_2I_2 & + (Z_2 + Z_3 + Z_5)I_3 & & & -Z_5I_4 & = -E_2 \\
 -Z_6I_1 & +0 & & -Z_5I_3 & + (Z_5 + Z_6 + Z_7)I_4 & & = 0
 \end{array}$$

Applying determinants:

$$I_{R_1} = I_{80\Omega} = 0.681 \text{ A } \angle -162.9^\circ$$

10.



Source Conversion:

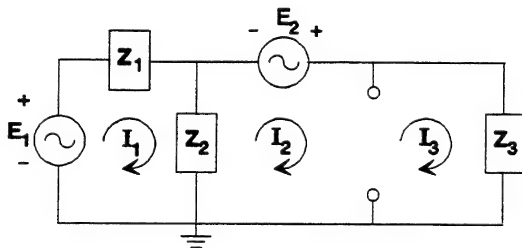
$$\begin{aligned}
 E &= (I \angle \theta)(R_p \angle 0^\circ) \\
 &= (50 \text{ I})(40 \text{ k}\Omega \angle 0^\circ) \\
 &= 2 \times 10^6 \text{ I } \angle 0^\circ \\
 Z_1 &= R_s = R_p = 40 \text{ k}\Omega \angle 0^\circ \\
 Z_2 &= -j0.2 \text{ k}\Omega \\
 Z_3 &= 8 \text{ k}\Omega \angle 0^\circ \\
 Z_4 &= 4 \text{ k}\Omega \angle 90^\circ
 \end{aligned}$$

$$\begin{aligned}
 I_1(Z_1 + Z_2 + Z_3) - Z_3I_2 &= -E \\
 I_2(Z_3 + Z_4) - Z_3I_1 &= 0
 \end{aligned}$$

$$\begin{aligned}
 (Z_1 + Z_2 + Z_3)I_1 - Z_3I_2 &= -E \\
 -Z_3I_1 + (Z_3 + Z_4)I_2 &= 0
 \end{aligned}$$

$$I_L = I_2 = \frac{-Z_3E}{(Z_1 + Z_2 + Z_3)(Z_3 + Z_4) - Z_3^2} = 42.91 \text{ I } \angle 149.31^\circ$$

12.



$$\begin{aligned}
 E_1 &= 5 \text{ V } \angle 0^\circ \\
 E_2 &= 20 \text{ V } \angle 0^\circ \\
 Z_1 &= 2.2 \text{ k}\Omega \angle 0^\circ \\
 Z_2 &= 5 \text{ k}\Omega \angle 90^\circ \\
 Z_3 &= 10 \text{ k}\Omega \angle 0^\circ \\
 I &= 4 \text{ mA } \angle 0^\circ
 \end{aligned}$$

$$\begin{aligned}
 E_1 - I_1Z_1 - Z_2(I_1 - I_2) &= 0 \\
 -Z_2(I_2 - I_1) + E_2 - I_3Z_3 &= 0
 \end{aligned}$$

$$I_3 - I_2 = I$$

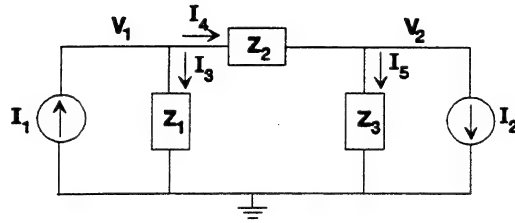
Substituting, we obtain:

$$\begin{aligned}
 I_1(Z_1 + Z_2) - I_2Z_2 &= E_1 \\
 I_1Z_2 - I_2(Z_2 + Z_3) &= IZ_3 - E_2
 \end{aligned}$$

Determinants:

$$\begin{aligned}
 I_1 &= 1.39 \text{ mA } \angle -126.48^\circ, I_2 = 1.341 \text{ mA } \angle -10.56^\circ, I_3 = 2.693 \text{ mA } \angle -174.8^\circ \\
 I_{10\text{k}\Omega} &= I_3 = 2.693 \text{ mA } \angle -174.8^\circ
 \end{aligned}$$

14. a.



$$\begin{aligned} Z_1 &= 4 \, \Omega \angle 0^\circ \\ Z_2 &= 5 \, \Omega \angle 90^\circ \\ Z_3 &= 2 \, \Omega \angle -90^\circ \\ I_1 &= 3 \, \text{A} \angle 0^\circ \\ I_2 &= 5 \, \text{A} \angle 30^\circ \end{aligned}$$

$$I_1 = I_3 + I_4$$

$$I_1 = \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} \Rightarrow V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_2} \right] - V_2 \left[\frac{1}{Z_2} \right] = I_1$$

$$\text{or } V_1[Y_1 + Y_2] - V_2[Y_2] = I_1$$

$$I_4 = I_5 + I_2$$

$$\frac{V_1 - V_2}{Z_2} = \frac{V_2}{Z_3} + I_2 \Rightarrow V_2 \left[\frac{1}{Z_2} + \frac{1}{Z_3} \right] - V_1 \left[\frac{1}{Z_2} \right] = -I_2$$

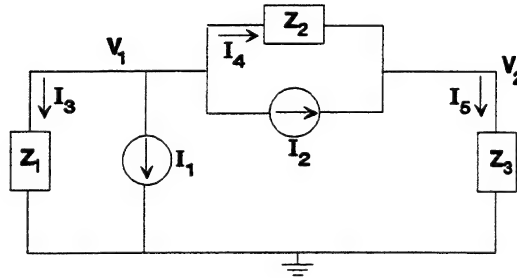
$$\text{or } V_2[Y_2 + Y_3] - V_1[Y_2] = -I_2$$

$$\begin{aligned} [Y_1 + Y_2]V_1 - Y_2V_2 &= I_1 \\ -Y_2V_1 + [Y_2 + Y_3]V_2 &= -I_2 \end{aligned}$$

$$V_1 = \frac{[Y_2 + Y_3]I_1 - Y_2I_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} = 14.68 \, \text{V} \angle 68.89^\circ$$

$$V_2 = \frac{-[Y_2 + Y_3]I_2 + Y_2I_1}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} = 12.97 \, \text{V} \angle 155.88^\circ$$

b.



$$\begin{aligned} Z_1 &= 3 \, \Omega + j4 \, \Omega = 5 \, \Omega \angle 53.13^\circ \\ Z_2 &= 2 \, \Omega \angle 0^\circ \\ Z_3 &= 6 \, \Omega \angle 0^\circ \parallel 8 \, \Omega \angle -90^\circ \\ &= 4.8 \, \Omega \angle -36.87^\circ \\ I_1 &= 0.6 \, \text{A} \angle 20^\circ \\ I_2 &= 4 \, \text{A} \angle 80^\circ \end{aligned}$$

$$0 = I_1 + I_3 + I_4 + I_2$$

$$0 = I_1 + \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} + I_2$$

$$V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_2} \right] - V_2 \left[\frac{1}{Z_2} \right] = -I_1 - I_2$$

$$\text{or } V_1[Y_1 + Y_2] - V_2[Y_2] = -I_1 - I_2$$

$$I_2 + I_4 = I_5$$

$$I_2 + \frac{V_1 - V_2}{Z_2} = \frac{V_2}{Z_3}$$

$$V_2 \left[\frac{1}{Z_2} + \frac{1}{Z_3} \right] - V_1 \left[\frac{1}{Z_2} \right] = +I_2$$

$$\text{or } V_2[Y_2 + Y_3] - V_1[Y_2] = I_2$$

$$\text{and } [Y_1 + Y_2]V_1 - Y_2V_2 = -I_1 - I_2$$

$$-Y_2V_1 + [Y_2 + Y_3]V_2 = I_2$$

Applying determinants:

$$V_1 = \frac{-[Y_2 + Y_3][I_1 + I_2] + Y_2I_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} = 5.12 \text{ V } \angle -79.36^\circ$$

$$V_2 = \frac{Y_1I_2 - I_1Y_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} = 2.71 \text{ V } \angle 39.96^\circ$$

$$16. \quad I = \frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2}$$

$$0 = \frac{V_2 - V_1}{Z_2} + \frac{V_2}{Z_3} + \frac{V_2 - E}{Z_4}$$

$$Z_1 = 2 \Omega \angle 0^\circ$$

$$Z_2 = 20 \Omega + j 20 \Omega$$

$$Z_3 = 10 \Omega \angle -90^\circ$$

$$Z_4 = 10 \Omega \angle 0^\circ$$

$$I = 6 \text{ A } \angle 0^\circ$$

$$E = 30 \text{ V } \angle 0^\circ$$

Rearranging:

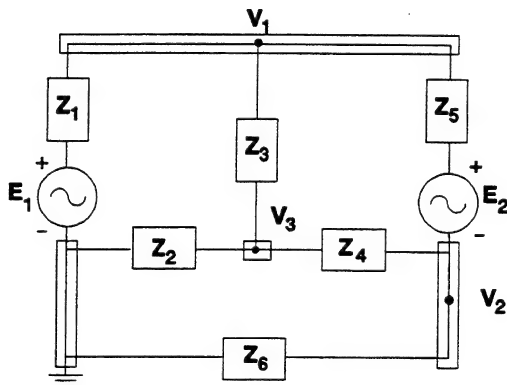
$$V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_2} \right] - \frac{1}{Z_2} V_2 = I$$

$$\frac{-V_1}{Z_2} + V_2 \left[\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4} \right] = \frac{E}{Z_4}$$

Determinants and substituting:

$$V_1 = 11.74 \text{ V } \angle -4.611^\circ, V_2 = 22.53 \text{ V } \angle -36.48^\circ$$

18.



$$Z_1 = 5 \Omega \angle 0^\circ$$

$$Z_2 = 6 \Omega \angle -90^\circ$$

$$Z_3 = 5 \Omega \angle 90^\circ$$

$$Z_4 = 4 \Omega \angle 0^\circ$$

$$Z_5 = 4 \Omega \angle 0^\circ$$

$$Z_6 = 6 \Omega + j8 \Omega$$

$$E_1 = 20 \text{ V } \angle 0^\circ$$

$$E_2 = 40 \text{ V } \angle 60^\circ$$

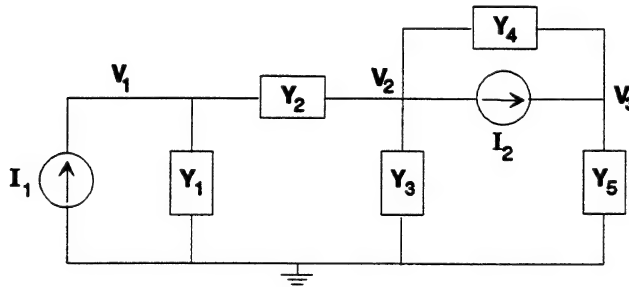
$$\begin{aligned}\text{node } V_1: \quad & \frac{V_1 - E_1}{Z_1} + \frac{V_1 - V_3}{Z_3} + \frac{V_1 - E_2 - V_2}{Z_5} = 0 \\ \text{node } V_2: \quad & \frac{V_2 + E_2 - V_1}{Z_5} + \frac{V_2 - V_3}{Z_4} + \frac{V_2}{Z_6} = 0 \\ \text{node } V_3: \quad & \frac{V_3}{Z_2} + \frac{V_3 - V_1}{Z_3} + \frac{V_3 - V_2}{Z_4} = 0\end{aligned}$$

Rearranging:

$$\begin{aligned}V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_3} + \frac{1}{Z_5} \right] - \frac{V_2}{Z_5} - \frac{V_3}{Z_3} &= \frac{E_1}{Z_1} + \frac{E_2}{Z_5} \\ V_2 \left[\frac{1}{Z_5} + \frac{1}{Z_4} + \frac{1}{Z_6} \right] - \frac{V_1}{Z_5} - \frac{V_3}{Z_4} &= -\frac{E_2}{Z_5} \\ V_3 \left[\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4} \right] - \frac{V_1}{Z_3} - \frac{V_2}{Z_4} &= 0\end{aligned}$$

Determinants: $V_1 = 5.839 \text{ V } \angle 29.4^\circ$, $V_2 = 28.06 \text{ V } \angle -89.15^\circ$, $V_3 = 31.96 \text{ V } \angle -77.6^\circ$

20. a.



$$\begin{aligned}Y_1 &= \frac{1}{4 \Omega \angle 0^\circ} \\ &= 0.25 \text{ S } \angle 0^\circ\end{aligned}$$

$$\begin{aligned}Y_2 &= \frac{1}{1 \Omega \angle 90^\circ} \\ &= 1 \text{ S } \angle -90^\circ\end{aligned}$$

$$\begin{aligned}Y_3 &= \frac{1}{5 \Omega \angle 0^\circ} \\ &= 0.2 \text{ S } \angle 0^\circ\end{aligned}$$

$$\begin{aligned}Y_4 &= \frac{1}{4 \Omega \angle -90^\circ} \\ &= 0.25 \text{ S } \angle 90^\circ\end{aligned}$$

$$\begin{aligned}Y_5 &= \frac{1}{8 \Omega \angle 90^\circ} \\ &= 0.125 \text{ S } \angle -90^\circ\end{aligned}$$

$$\begin{aligned}I_1 &= 2 \text{ A } \angle 30^\circ \\ I_2 &= 3 \text{ A } \angle 150^\circ\end{aligned}$$

$$\begin{aligned}V_1[Y_1 + Y_2] - Y_2V_2 &= I_1 \\ V_2[Y_2 + Y_3 + Y_4] - Y_2V_1 - Y_4V_3 &= -I_2 \\ V_3[Y_4 + Y_5] - Y_4V_2 &= I_2\end{aligned}$$

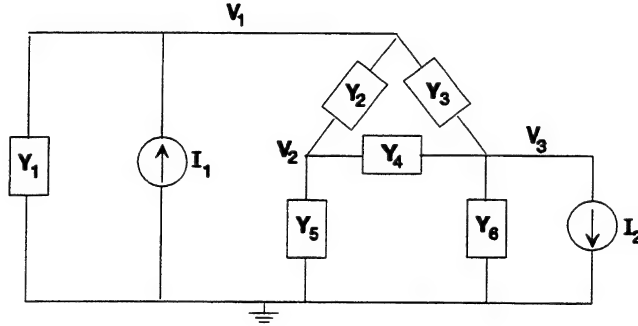
$$\begin{aligned}[Y_1 + Y_2]V_1 &\quad - Y_2V_2 &\quad + 0 &= I_1 \\ -Y_2V_1 &+ [Y_2 + Y_3 + Y_4]V_2 &\quad - Y_4V_3 &= -I_2 \\ 0 &\quad - Y_4V_2 &+ [Y_4 + Y_5]V_3 &= I_2\end{aligned}$$

$$\begin{aligned}V_1 &= \frac{I_1[(Y_2 + Y_3 + Y_4)(Y_4 + Y_5) - Y_4^2] - I_2[Y_2Y_5]}{[Y_1 + Y_2][(Y_2 + Y_3 + Y_4)(Y_4 + Y_5) - Y_4^2] - Y_2^2(Y_4 + Y_5)} \\ &= 5.74 \text{ V } \angle 122.76^\circ\end{aligned}$$

$$V_2 = \frac{I_1 Y_2 (Y_4 + Y_5) - I_2 Y_5 (Y_1 + Y_2)}{Y_\Delta} = 4.04 \text{ V } \angle 145.03^\circ$$

$$V_3 = \frac{I_2 [(Y_1 + Y_2)(Y_3 + Y_4) - Y_2^2] - Y_2 Y_4 I_1}{Y_\Delta} = 25.94 \text{ V } \angle 78.07^\circ$$

b.



$$Y_1 = \frac{1}{4 \Omega \angle 0^\circ} = 0.25 \text{ S } \angle 0^\circ$$

$$Y_2 = \frac{1}{6 \Omega \angle 0^\circ} = 0.167 \text{ S } \angle 0^\circ$$

$$Y_3 = \frac{1}{8 \Omega \angle 0^\circ} = 0.125 \text{ S } \angle 0^\circ$$

$$Y_4 = \frac{1}{2 \Omega \angle -90^\circ} = 0.5 \text{ S } \angle 90^\circ$$

$$Y_5 = \frac{1}{5 \Omega \angle 90^\circ} = 0.2 \text{ S } \angle -90^\circ$$

$$Y_6 = \frac{1}{4 \Omega \angle 90^\circ} = 0.25 \text{ S } \angle -90^\circ$$

$$I_1 = 4 \text{ A } \angle 0^\circ$$

$$I_2 = 6 \text{ A } \angle 90^\circ$$

$$\begin{aligned} V_1[Y_1 + Y_2 + Y_3] - Y_2 V_2 - Y_3 V_3 &= I_1 \\ V_2[Y_2 + Y_4 + Y_5] - Y_2 V_1 - Y_4 V_3 &= 0 \\ V_3[Y_3 + Y_4 + Y_6] - Y_3 V_1 - Y_4 V_2 &= -I_2 \end{aligned}$$

$$\begin{aligned} [Y_1 + Y_2 + Y_3]V_1 &\quad - Y_2 V_2 &\quad - Y_3 V_3 &= I_1 \\ -Y_2 V_1 + [Y_2 + Y_4 + Y_5]V_2 &\quad - Y_4 V_3 &= 0 \\ -Y_3 V_1 &\quad - Y_4 V_2 + [Y_3 + Y_4 + Y_6]V_3 &= -I_2 \end{aligned}$$

$$V_1 = \frac{I_1 [(Y_2 + Y_4 + Y_5)(Y_3 + Y_4 + Y_6) - Y_4^2] - I_2 [Y_2 Y_4 + Y_3(Y_3 + Y_4 + Y_5)]}{Y_\Delta = (Y_1 + Y_2 + Y_3)[(Y_2 + Y_4 + Y_5)(Y_3 + Y_4 + Y_6) - Y_4^2] - Y_2[Y_2(Y_3 + Y_4 + Y_6) + Y_3 Y_4] - Y_3[Y_2 Y_4 + Y_3(Y_2 + Y_4 + Y_5)]}$$

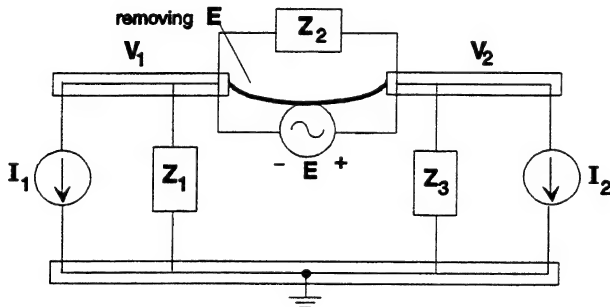
$$= 15.13 \text{ V } \angle 1.29^\circ$$

$$V_2 = \frac{I_1 [(Y_2)(Y_3 + Y_4 + Y_6) + Y_3 Y_4] + I_2 [Y_4(Y_1 + Y_2 + Y_3) - Y_2 Y_3]}{Y_\Delta} = 17.24 \text{ V } \angle 3.73^\circ$$

$$V_3 = \frac{I_1 [(Y_3)(Y_2 + Y_4 + Y_5) + Y_2 Y_4] + I_2 [Y_2^2 - (Y_1 + Y_2 + Y_3)(Y_2 + Y_4 + Y_5)]}{Y_\Delta}$$

$$= 10.59 \text{ V } \angle -0.11^\circ$$

22.



$$Z_1 = 1 \text{ k}\Omega \angle 0^\circ$$

$$Z_2 = 2 \text{ k}\Omega \angle 90^\circ$$

$$Z_3 = 3 \text{ k}\Omega \angle -90^\circ$$

$$I_1 = 12 \text{ mA } \angle 0^\circ$$

$$I_2 = 4 \text{ mA } \angle 0^\circ$$

$$E = 10 \text{ V } \angle 0^\circ$$

$$\begin{aligned}\Sigma I_i &= \Sigma I_o \\ 0 &= I_1 + \frac{V_1}{Z_1} + \frac{V_2}{Z_3} + I_2 \\ \text{and } \frac{V_1}{Z_1} + \frac{V_2}{Z_3} &= -I_1 - I_2 \\ \text{with } V_2 - V_1 &= E\end{aligned}$$

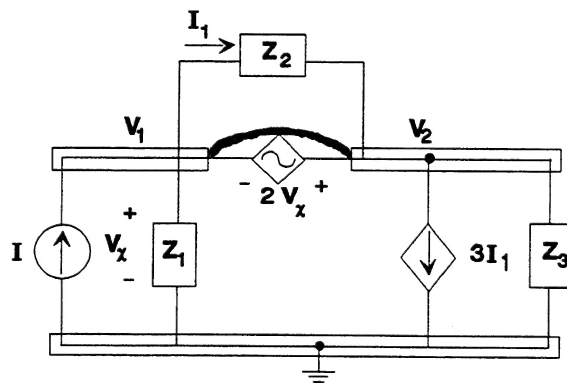
Substituting and rearranging:

$$V_1 \left[\frac{1}{Z_1} + \frac{1}{Z_3} \right] = -I_1 - I_2 - \frac{E}{Z_3}$$

and solving for V_1 :

$$\begin{aligned}V_1 &= 15.4 \text{ V } \angle 178.2^\circ \\ \text{with } V_2 &= 5.414 \text{ V } \angle 174.87^\circ\end{aligned}$$

24.



$$\begin{aligned}Z_1 &= 2 \text{ k}\Omega \angle 0^\circ \\ Z_2 &= 1 \text{ k}\Omega \angle 0^\circ \\ Z_3 &= 1 \text{ k}\Omega \angle 0^\circ \\ I &= 5 \text{ mA } \angle 0^\circ\end{aligned}$$

$$\begin{aligned}V_1: I &= \frac{V_1}{Z_1} + 3I_1 + \frac{V_2}{Z_3} \\ \text{with } I_1 &= \frac{V_1 - V_2}{Z_2} \\ \text{and } V_2 - V_1 &= 2V_x = 2V_1 \text{ or } V_2 = 3V_1\end{aligned}$$

Substituting with result in:

$$V_1 \left[\frac{1}{Z_1} + \frac{3}{Z_2} \right] + 3 V_1 \left[\frac{1}{Z_3} - \frac{3}{Z_2} \right] = I$$

$$\text{or } V_1 \left[\frac{1}{Z_1} - \frac{6}{Z_2} + \frac{3}{Z_3} \right] = I$$

$$\begin{aligned}\text{and } V_1 &= -2 \text{ V } \angle 0^\circ \\ \text{with } V_2 &= -6 \text{ V } \angle 0^\circ\end{aligned}$$

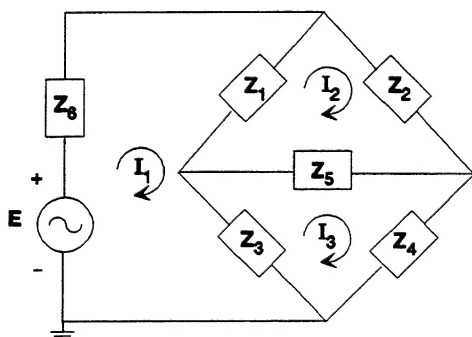
26. a. yes

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

$$\frac{5 \times 10^3 \angle 0^\circ}{2.5 \times 10^3 \angle 90^\circ} = \frac{8 \times 10^3 \angle 0^\circ}{4 \times 10^3 \angle 90^\circ}$$

$$2 \angle -90^\circ = 2 \angle -90^\circ \text{ (balanced)} \checkmark$$

b. $Z_1 = 5 \text{ k}\Omega \angle 0^\circ$, $Z_2 = 8 \text{ k}\Omega \angle 0^\circ$
 $Z_3 = 2.5 \text{ k}\Omega \angle 90^\circ$, $Z_4 = 4 \text{ k}\Omega \angle 90^\circ$
 $Z_5 = 5 \text{ k}\Omega \angle -90^\circ$, $Z_6 = 1 \text{ k}\Omega \angle 0^\circ$



$$\begin{aligned} I_1[Z_1 + Z_3 + Z_6] - Z_1 I_2 - Z_3 I_3 &= E \\ I_2[Z_1 + Z_2 + Z_5] - Z_1 I_1 - Z_5 I_3 &= 0 \\ I_3[Z_3 + Z_4 + Z_5] - Z_3 I_1 - Z_5 I_2 &= 0 \end{aligned}$$

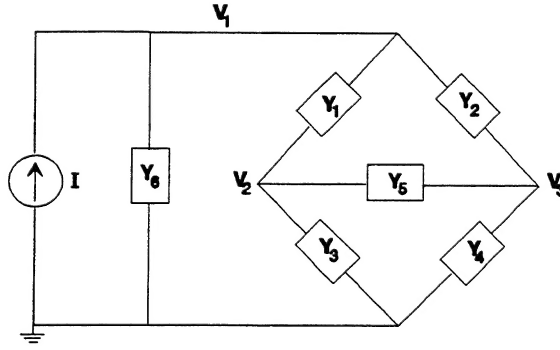
$$\begin{aligned} [Z_1 + Z_3 + Z_6]I_1 &\quad - Z_1 I_2 &\quad - Z_3 I_3 &= E \\ -Z_1 I_1 + [Z_1 + Z_2 + Z_5]I_2 &\quad &\quad - Z_5 I_3 &= 0 \\ -Z_3 I_1 &\quad - Z_5 I_2 + [Z_3 + Z_4 + Z_5]I_3 &\quad &= 0 \end{aligned}$$

$$I_2 = \frac{E[Z_1(Z_3 + Z_4 + Z_5) + Z_3 Z_5]}{Z_\Delta = (Z_1 + Z_3 + Z_6)(Z_1 + Z_2 + Z_5)(Z_3 + Z_4 + Z_5) - Z_5^2 - Z_1[Z_1(Z_3 + Z_4 + Z_5) - Z_3 Z_5] - Z_3[Z_1 Z_5 + Z_3(Z_1 + Z_2 + Z_5)]}$$

$$I_3 = \frac{E[Z_1 Z_5 + Z_3(Z_1 + Z_2 + Z_5)]}{Z_\Delta}$$

$$I_{Z_5} = I_2 - I_3 = \frac{E[Z_1 Z_4 - Z_3 Z_2]}{Z_\Delta} = \frac{E[20 \times 10^6 \angle 90^\circ - 20 \times 10^6 \angle 90^\circ]}{Z_\Delta} = 0 \text{ A}$$

c.



$$\begin{aligned} Y_1[V_1 + Y_2 + Y_6] - Y_1V_2 - Y_2V_3 &= I \\ V_2[Y_1 + Y_3 + Y_5] - Y_1V_1 - Y_5V_3 &= 0 \\ V_3[Y_2 + Y_4 + Y_5] - Y_2V_1 - Y_5V_2 &= 0 \end{aligned}$$

$$\begin{aligned} [Y_1 + Y_2 + Y_6]V_1 - Y_1V_2 - Y_2V_3 &= I \\ -Y_1V_1 + [Y_1 + Y_3 + Y_5]V_2 - Y_5V_3 &= 0 \\ -Y_2V_1 - Y_5V_2 + [Y_2 + Y_4 + Y_5]V_3 &= 0 \end{aligned}$$

$$V_2 = \frac{I[Y_1(Y_2 + Y_4 + Y_5) + Y_2Y_3]}{Y_\Delta = (Y_1 + Y_2 + Y_6)[(Y_1 + Y_3 + Y_5)(Y_2 + Y_4 + Y_5) - Y_5^2] - Y_1[Y_1(Y_2 + Y_4 + Y_5) + Y_2Y_3] - Y_2[Y_1Y_5 + Y_2(Y_1 + Y_3 + Y_5)]}$$

$$V_3 = \frac{I[(Y_1Y_5 + Y_2(Y_1 + Y_3 + Y_5))]}{Y_\Delta}$$

$$\begin{aligned} V_{Z_5} = V_2 - V_3 &= \frac{I[Y_1Y_4 - Y_2Y_3]}{Y_\Delta} = \frac{I[0.05 \times 10^{-3} \angle -90^\circ - 0.05 \times 10^{-3} \angle -90^\circ]}{Y_\Delta} \\ &= 0 \text{ V} \end{aligned}$$

$$I = \frac{E_s}{R_s} = \frac{10 \text{ V} \angle 0^\circ}{1 \text{ k}\Omega \angle 0^\circ} = 10 \text{ mA} \angle 0^\circ$$

$$Y_1 = \frac{1}{5 \text{ k}\Omega \angle 0^\circ} = 0.2 \text{ mS} \angle 0^\circ$$

$$Y_2 = \frac{1}{8 \text{ k}\Omega \angle 0^\circ} = 0.125 \text{ mS} \angle 0^\circ$$

$$Y_3 = \frac{1}{2.5 \text{ k}\Omega \angle 90^\circ} = 0.4 \text{ mS} \angle -90^\circ$$

$$Y_4 = \frac{1}{4 \text{ k}\Omega \angle 90^\circ} = 0.25 \text{ mS} \angle -90^\circ$$

$$Y_5 = \frac{1}{5 \text{ k}\Omega \angle -90^\circ} = 0.2 \text{ mS} \angle 90^\circ$$

$$Y_6 = \frac{1}{1 \text{ k}\Omega \angle 0^\circ} = 1 \text{ mS} \angle 0^\circ$$

28. $Z_1Z_4 = Z_3Z_2$

$$(R_1 - jX_C)(R_x + jX_{L_x}) = R_3R_2 \quad X_C = \frac{1}{\omega C} = \frac{1}{(10^3 \text{ rad/s})(1 \mu\text{F})} = 1 \text{ k}\Omega$$

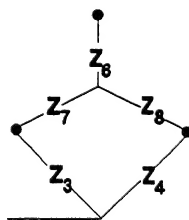
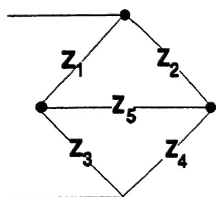
$$(1 \text{ k}\Omega - j1 \text{ k}\Omega)(R_x + jX_{L_x}) = (0.1 \text{ k}\Omega)(0.1 \text{ k}\Omega) = 10 \text{ k}\Omega$$

$$\text{and } R_x + jX_{L_x} = \frac{10 \times 10^3 \Omega}{1 \times 10^3 - j1 \times 10^3} = \frac{10 \times 10^3}{1.414 \times 10^3 \angle -45^\circ} = 5 \Omega + j5 \Omega$$

$$\therefore R_x = 5 \Omega, L_x = \frac{X_{L_x}}{\omega} = \frac{5 \Omega}{10^3 \text{ rad/s}} = 5 \text{ mH}$$

30. Apply Eq. 17.6.

32. a.



$$\begin{aligned} Z_1 &= 8 \Omega \angle -90^\circ = -j8 \Omega \\ Z_2 &= 4 \Omega \angle 90^\circ = +j4 \Omega \\ Z_3 &= 8 \Omega \angle 90^\circ = +j8 \Omega \\ Z_4 &= 6 \Omega \angle -90^\circ = -j6 \Omega \\ Z_5 &= 5 \Omega \angle 0^\circ \end{aligned}$$

$$Z_6 = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_5} = 5 \Omega \angle 38.66^\circ$$

$$Z_7 = \frac{Z_1 Z_5}{Z_1 + Z_2 + Z_5} = 6.25 \Omega \angle -51.34^\circ$$

$$Z_8 = \frac{Z_2 Z_5}{Z_1 + Z_2 + Z_5} = 3.125 \Omega \angle 128.66^\circ$$

$$Z' = Z_7 + Z_3 = 3.9 \Omega + j3.12 \Omega = 4.99 \Omega \angle 38.66^\circ$$

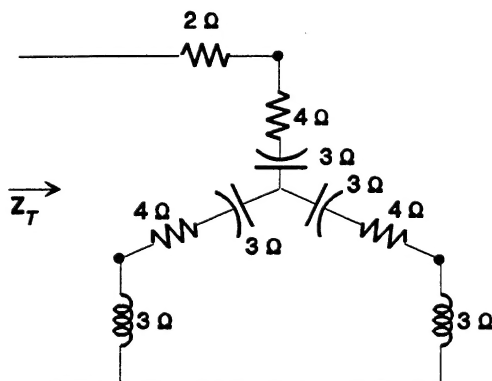
$$Z'' = Z_8 + Z_4 = -1.95 \Omega - j3.56 \Omega = 4.06 \Omega \angle -118.71^\circ$$

$$Z' \parallel Z'' = 10.13 \Omega \angle -67.33^\circ = 3.90 \Omega - j9.35 \Omega$$

$$Z_T = Z_6 + Z' \parallel Z'' = 7.80 \Omega - j6.23 \Omega = 9.98 \Omega \angle -38.61^\circ$$

$$I = \frac{E}{Z_T} = \frac{120 \text{ V} \angle 0^\circ}{9.98 \Omega \angle -38.61^\circ} = 12.02 \text{ A} \angle 38.61^\circ$$

b. $Z_Y = \frac{Z_\Delta}{3} = \frac{12 \Omega - j9 \Omega}{3} = 4 \Omega - j3 \Omega$



$$\begin{aligned} Z_T &= 2 \Omega + 4 \Omega + j3 \Omega + [4 \Omega - j3 \Omega + j3 \Omega] \parallel [4 \Omega - j3 \Omega + j3 \Omega] \\ &= 6 \Omega - j3 \Omega + 2 \Omega \\ &= 8 \Omega - j3 \Omega = 8.544 \Omega \angle -20.56^\circ \end{aligned}$$

$$I = \frac{E}{Z_T} = \frac{60 \text{ V} \angle 0^\circ}{8.544 \Omega \angle -20.56^\circ} = 7.02 \text{ A} \angle 20.56^\circ$$